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Effective viscosity in a Lévy-walk model for turbulent channel flow

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We show how in the Lévy-walk model for turbulent channel flow one can measure an effective viscosity in a two-dimensional lattice-gas implementation. We study the dependence of effective viscosity on the exponent characterizing the distribution of distances over which momentum is exchanged and on system size.

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INTRODUCTION

Lévy walks were proposed for a description of enhanced diffusion in developed turbulence [1] and more recently discussed in the context of "strange" kinetics of chaotic Hamiltonian systems [2]. What characterizes Lévy walks is a distribution of step sizes which decays algebraically for long distances and a time associated with each step, varying with distance.

In Refs. [3] and [4] that basic idea was implemented for the type of hexagonal lattice gas [5] which, upon coarse graining, has been shown to lead to a description of fluid flow. The flow studied was turbulent channel flow in two dimensions, and both average velocity profile and Reynolds stresses were described and discussed.

The implementation of Lévy walks in a lattice gas is similar to a closure approximation for the Navier-Stokes equation of turbulent flows. The simplest closure approximation amounts to replacing in the equation for the average velocity the molecular kinematic viscosity by an effective one. Now, for turbulent channel flow, this effective viscosity is proportional to the product of the characteristic velocity and Prandtl's mixing length. The mixing length itself refers to momentum exchanges, as does the Lévy-walk implementation at the lattice-gas level [3]. The question is whether in the latter approach an effective viscosity can be defined and similarly related to the average length of Lévy walks. In answer, we here propose a method to measure an effective viscosity and study its relation to the average length of Lévy walks. The method consists in measuring the decay of average velocity of some initial flow while implementing the Lévy-walk exchanges all during the decay. It turns out that the flow decays exponentially with a decay constant which we relate to an effective viscosity. As it should, this effective viscosity, which is larger than the molecular one, decreases when the average distance over which exchanges take place gets smaller.

As in the study of driven channel flow, we realize the Lévy walk in the two-dimensional 6-bit lattice gas with energy and momentum conserving collision rules [3,5] by exchanging the populations of sites separated by distance l, perpendicular to the mean flow direction. l is drawn with a probability

$$p(l)=Al^{-\alpha} \tag{1}$$

where A is a normalization constant. Moreover, the number of exchanges depends linearly on l, such that there are fewer exchanges the greater the distance [3]. The exponent α is the main parameter in our study [3]. The exchanges have the effect of flattening the velocity profile and enhancing the transport of momentum to the walls of the channel, thus increasing the viscosity of the fluid.

For the simulations we generate a channel filled with lattice-gas fluid with a Poiseuille velocity profile, with periodic boundary conditions connecting the open ends, and with bounce-back boundary conditions defining the channel walls. The algorithm is implemented using multispin coding for system sizes of 250×200 and 250×512.

RESULTS

Starting from an initial velocity field pointing along one of the sides of the rectangular system, we measure the decay of the velocity averaged over the whole system. Results do not depend on whether in the initial flow a Lévy walk is implemented or whether it is laminar. In the latter case the exchanges flatten the profile in a time short compared to the total decay time, and from then one the profile decays uniformly. What is crucial is that the Lévy-walk exchanges continue while the flow velocity is decaying. Since we find that the decay is exponential in time (Fig. 1), we extract from the decay constant τ an effective viscosity ν_{eff} related to it by $\nu_{\text{eff}}=L^2/\tau\pi^2$, where L is the dimension of the system transverse to the initial flow and the factor of π is present because of the analogous formula in the laminar case.

Our principal result is the curve shown in Fig. 2 for the effective viscosity as a function of α. This viscosity is a factor of 2.5 larger than the molecular one at small α for a system size $L=200$. ν_{eff} decreases as α increases, because then the exchange of momenta takes place over smaller and smaller distances [cf. Eq. (1)]. When α gets too large the exchanges are over such short distances that no significant momentum exchange takes place. As a result ν_{eff} tends towards the molecular viscosity. In Fig. 2 we also show the behavior of the mean exchange length $\bar{l}$

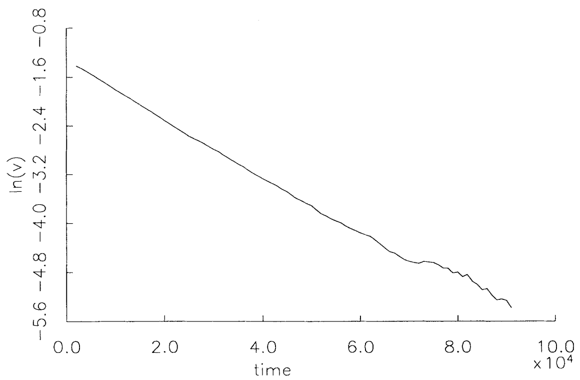

FIG. 1. Plot of decaying velocity as a function of time. The logarithm of the velocity is plotted. The figure corresponds to $L=512$ and $\alpha=1.1$.

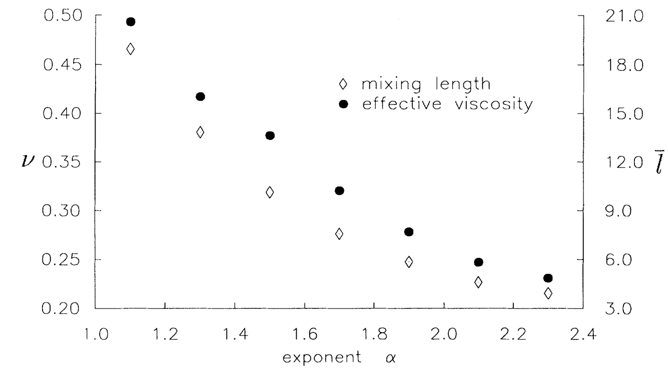

FIG. 2. Plot of effective viscosity and mixing length as a function of α. Symbol sizes indicate error margins. Note that the viscosity for laminar flow is 0.186. The figure corresponds to $L=200$.

with α. The behavior is similar to that of ν_{eff}, suggesting a relation

$$\nu_{\text{eff}}=v^{*}\bar{l} \qquad (2)$$

with ν_{eff} the effective viscosity, v^* a quantity with the dimension of velocity, and $\bar{l}$ the mean exchange length. (The mean length is not sharply defined because fluctuations are large due to the slow tapering of the corresponding probability distribution.) Equation (2) expresses a bulk relationship since it is obtained from averaging the flow through the whole system.

The value of v^* depends on α and somewhat on L. At low α, where the channel because of long-distance momentum exchanges is the "most turbulent," values are 0.026 and 0.030 at, respectively, $\alpha=1.1$ and 1.3 for $L=200$ and 0.033 at $\alpha=1.1$ for $L=512$. These values of v^* are remarkably close to that obtained from the average velocity profile in driven turbulent channel flow for which the characteristic velocity was determined to be 0.02 [3]. The relationship between ν_{eff} and v^* is an element of consistency which is included in the usual closure approximation for the Navier-Stokes equation [6].

The mean $\bar{l}$ increases with the size of the system, asymptotically as $\bar{l}\sim L^{2-\alpha}$. The effective viscosity increases correspondingly, exactly as $\bar{l}$ if one neglects the small system size dependence of v^* discussed in the preceding paragraph. We have checked this from our numerical results. The dependence on L is expected since the viscosity increase in turbulent channel flow is due to momentum exchanges over large distances, with larger momenta conveyed from far away to the vicinity of the wall where they decay.

DISCUSSION

We have proposed a "trick" to measure the effective viscosity of a lattice gas made turbulent through Lévy walks for the case of channel flow. It is a function of the exponent which determines the distribution of distances over which Lévy walks occur and also of system size. Its behavior is sensible, as it is proportional to the average length of the walks, with a proportionality constant close in value to the characteristic velocity of sustained turbulent channel flow. The analogy with the usual closure approximation mentioned in the Introduction is thus established. A final remark is in order: The trick for measuring ν_{eff} consists in maintaining momentum exchanges at the same level, with the same characteristic exponent α during velocity decay. This way of proceeding is not relevant to a description of the actual physics of a decaying turbulent channel flow, for which the Lévy walk itself would presumably have to evolve with the changing characteristic velocity of the logarithmic profile [3]. In the decay itself, as investigated here, the flat velocity profile decays exponentially, uniformly in space. The question may be asked whether if one had a realistic description of the decaying turbulent flow, behavior other than exponential would emerge. We have implemented a number of simple modifications of the basic Lévy-walk algorithm in order to model the decay more realistically, such as making exchanges depend not only on distance, but also on the values of momenta involved, without ever finding behavior other than exponential for the decay of bulk velocity. There is of course no reason to expect that the power-law decays of isotropic turbulence for velocity or velocity correlations [7], for which Loitsyanskii's law appears, is of any relevance to turbulent channel decay.

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[1] M. F. Shlesinger, B. J. West, and J. Klafter, Phys. Rev. Lett. **58**, 1100 (1987).
[2] M. F. Shlesinger, G. M. Zaslavsky, and J. Klafter, Nature **363**, 31 (1993).
[3] F. Hayot, Phys. Rev. A **43**, 806 (1991).
[4] F. Hayot, J. Stat. Phys. **68**, 34 (1992).
[5] U. Frisch, B. Hasslacher, and Y. Pomeau, Phys. Rev. Lett. **56**, 1505 (1986); S. Wolfram, J. Stat. Phys. **45**, 471 (1986).
[6] W. C. Reynolds, Annu. Rev. Fluid Mech. **8**, 183 (1976).
[7] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1959).